Sheet 5

1 What is meant by the curl of a vector field ? Give an example to explain it

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2 Compute the curl of the following vector fields

a) $\bar{F} = xy\bar{a}_x + 2yz\bar{a}_y - \bar{a}_z$

b) $\bar{F} = 2\bar{a}_r + \sin\phi\,\bar{a}_\phi - z\bar{a}_z$

c) $\bar{F} = r\bar{a}_r + \bar{a}_\theta + \sin\theta\,\bar{a}_\phi$

$$\left[\begin{array}{c} \nabla\times\bar{F} = -2y\bar{a}_x - x\bar{a}_z \\ \nabla\times\bar{F} = \frac{1}{\rho}\sin\phi\ \bar{a}_z \\ \nabla\times\bar{F} = \frac{2\cos\theta}{r}\bar{a}_r - \frac{\sin\theta}{r}\bar{a}_\theta + \frac{1}{r}\bar{a}_\phi \end{array}\right]$$

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3 The flow vector for a fluid flowing in a cylindrical pipe of a unit inner radius is given by

$$\bar{F} = \left[\frac{1-r}{1+r}\right]\bar{a}_z$$

Determine the curl of $\bar{F}$ at the axis and at the inner surface of the pipe. Sketch the profile of the curl over the pipe's cross section

$$\left[\nabla\times\bar{F} = \frac{2}{(1+r)^2}\bar{a}_\phi\right]$$

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4 Verify Stoke's theorem for a flat rectangular surface in the xy-plane bounded by [0,0,0] , [1,0,0] , [1,1,0] , [0,1,0] , when

a) $\bar{F} = 2\bar{a}_x + \bar{a}_y$

b) $\bar{F} = 2xy\bar{a}_x - y\bar{a}_z$

$$\left[\begin{array}{c} \oint \bar{F}\cdot\bar{dl} = 0 \\ \oint \bar{F}\cdot\bar{dl} = -1 \end{array}\right]$$

5 Determine the circulation of $\bar{F} = \bar{a}_r + \bar{a}_\theta + \bar{a}_\phi$ where the surface is defined by $r = 2$, $0 \leq \phi \leq \frac{\pi}{2}$ and $0 \leq \theta \leq \frac{\pi}{2}$. Then verify your result by using the curl

$$\left[\oint \bar{\boldsymbol{F}} \cdot \overline{\boldsymbol{dl}} = \boldsymbol{\pi}\right]$$

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6 Verify the following vector identities by direct expansion in a coordinate system of your choice

a) $\nabla \times \nabla f = 0$

b) $\nabla . \nabla \times \bar{F} = 0$

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7 Determine the circulation of the vector $\bar{F} = 5y\bar{a}_x + 3x\bar{a}_y - 2z\bar{a}_z$ around a closed contour from (1,1,0) to (1,4,0) to (2,1,0) , then verify Stoke’s theorem .

$$\left[\oint \bar{\boldsymbol{F}} \cdot \overline{\boldsymbol{dl}} = \boldsymbol{3}\right]$$
